A theoretical Framework for Qualitative Analysis of Large-scale Complex Systems

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Introduction

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Quantitative Analysis

❖ Advantages

  ❧ Description of Data in detail

  ❧ Precise local analysis

  ❧ Accurate data prediction

❖ Disadvantages

  ❧ Suitable for small-scale simple system

  ❧ Can not make us cognize things as whole

  ❧ Can not make relations among things clear

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Complex Systems

Qualitative Analysis

❖ Advantages

    ✍ Suitable for large-scale complex system

    ✍ Make us cognize things as whole

    ✍ Make relation among things clear

    ✍ Simple and convenient operation

❖ Difficulty

    ✍ Without analysis tool

    ✍ Without theoretical architecture

    ✍ Change of thinking mode

Algebraic System on Interval

1 **Definitions of Basic Concepts**

2 **Relations Between intervals**

3 **Operations on interval set**

4 **Algebraic System On Interval Set**

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Definitions of Basic Concepts

❖ Interval set S

  ℰ Basis of algebraic system

  ℰ Consists of intervals

  ℰ Covers some field

  ℰ Hierarchical structure

❖ Unitary Operations

  ℰ Convert an interval to other interval

  ℰ Serve for binary operations

  ℰ Applied to evaluate polynomial on interval

Interval Set

❖ Conception

  ✦ Every operation is defined on interval set

  ✦ Algebraic system consists of interval set and operations

  ✦ Contains all intervals which are discussed

  ✦ Consists of interval subsets hierarchically

  ✦ Covers all elements of some filed

❖ Denotation

  ✦ Denoted by S

Interval Subset

- ❖ Conception
  - ⚘ Subset of interval set S
  - ⚘ Each consists of some intervals which have same properties
  - ⚘ A interval subset may contain other interval subset
- ❖ Character
  - ⚘ Some interval subsets correspond with one or some scales
  - ⚘ Reflect the change of visual angle
- ❖ Utilities
  - ⚘ Reduce range of research
  - ⚘ Locate research at proper scale

Interval

❖ Conception

  ✧ Element of Interval set

  ✧ Belongs to some interval subset

  ✧ Describes a range covering many elements of some field

  ✧ Forms bottom layer of hierarchical structure of interval system

❖ Denotation

  ✧ Denoted by quaternary vector

    ✤ level component

    ✤ location component

    ✤ length component

    ✤ index component

Type of Interval

- ❖ Primitive Interval
  - ✍ Definition: index component of interval is zero
  - ✍ Has two types
    - ✄ Unit interval
      - ⚓ Definition: length component of interval is 1
      - ⚓ Character: the smallest interval on its level
    - ✄ Composite interval
      - ⚓ Definition: length component of interval is greater than 1
      - ⚓ Character: contain some unit intervals
- ❖ Non-primitive Interval
  - ✍ Definition: index component of interval is not zero
  - ✍ Derived from result of operation on primitive intervals
  - ✍ Represents the union of two primitive intervals

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Relations Between intervals

❖ Relation of equivalency
  ❧ Equivalence: an interval is equal to other interval
  ❧ No-more-than: an interval is no more than other interval
  ❧ Less-than: an interval is less than other interval

❖ Relation of order
  ❧ Intersecting: an interval intersects other interval
  ❧ Non-intersecting: an interval do not intersect other interval
  ❧ Join: an interval joins other interval
  ❧ Containing: an interval contains other interval
  ❧ Bordering-on: an interval borders on other interval

Unitary Operations(1)

- ❖ Positive-direction
  - ✦ Definition: adjust direction of interval to positive direction
  - ✦ Character: the result is an interval whose length component is positive integer
  - ✦ Interval set S is closed under this operation

- ❖ Absolute-value
  - ✦ Definition: assign zero to index component of interval
  - ✦ Character:
    - ✂ make a non-primitive interval be a primitive interval
    - ✂ One of operations that change type of interval
  - ✦ Interval set S is closed under this operation

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Unitary Operations(2)

- ❖ Complete-absolute
  - ❧ Definition: expand an interval to other interval whose index component is zero
  - ❧ Character:
    - ✕ Convert a non-primitive interval into a primitive interval
    - ✕ Another operation that changes type of interval
  - ❧ S is closed under this operation
- ❖ Jump-level
  - ❧ Definition: move interval from its level to other level
  - ❧ Character: convert an interval to other interval that is equal to this interval but whose level component is distinct
  - ❧ S is closed under this operation

Binary Operations(1)

- ❖ Union on interval
  - ❧ Definition: unite two intervals to produce a bigger one
  - ❧ Character:
    - ✄ the result is an interval whose index component is zero and length component is the sum of length components of two operands
    - ✄ An auxiliary operation
  - ❧ Properties
    - ✄ Closed: S equipped with operation union is closed
    - ✄ Associative: S is associative under operation union
    - ✄ Commutative: S is commutative under this operation
- ❖ System consisting of S and operation union
  - ❧ Be an algebraic system: since it is closed
  - ❧ Be a semi-group: since it satisfies associative law
  - ❧ Be a commutative semi-group: since it is commutative

Binary Operations(2)

- ❖ Addition on interval
  - ☙ Character
    - ✄ Be similar to conventional addition on number
    - ✄ Defined in interval set S
    - ✄ Operand is interval
  - ☙ Utility
    - ✄ One of important operations for constructing linear algebraic system
    - ✄ One of basic operations for fitting complex system
- ❖ Multiplication on interval
  - ☙ Character
    - ✄ Be similar to conventional multiplication on number
    - ✄ Defined in interval set S
    - ✄ operand is interval
  - ☙ Utility:
    - ✄ The other important operation for constructing linear algebraic system
    - ✄ The other basic operation for fitting complex system

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Algebraic System on S

❖ Conception
  ↬ Components
    ✄ Interval set S
    ✄ All operations defined on S
      ✚ Unitary Operations: defined as before
      ✚ Binary operations: defined as before
  ↬ Theoretical basis
    ✄ Each Operand of operation is interval of S
    ✄ Each operation is well-defined
    ✄ S is closed under each operation

❖ Utilities
  ↬ Constructs simple system to fit complex system
  ↬ Supports conversion of system between different scales

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Linear Algebraic System

1	**Linear Algebraic System on Interval Set**

2	**Additional on Interval**

3	**Multiplication on Interval**

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Linear Algebraic System on *S*

- ❖ Conception
  - ꙮ Components
    - ✖ Interval set *S*
    - ✖ Operations on *S*: addition-interval, multiplication-interval
  - ꙮ Character
    - ✖ Subsystem of algebraic system on *S* defined as before
    - ✖ Has particular relations between operation addition and operation multiplication
- ❖ Utilities
  - ꙮ Forms an operational architecture of large-scale
  - ꙮ Constructs a simple linear system to describe complex system

Addition on interval

❖ Conception
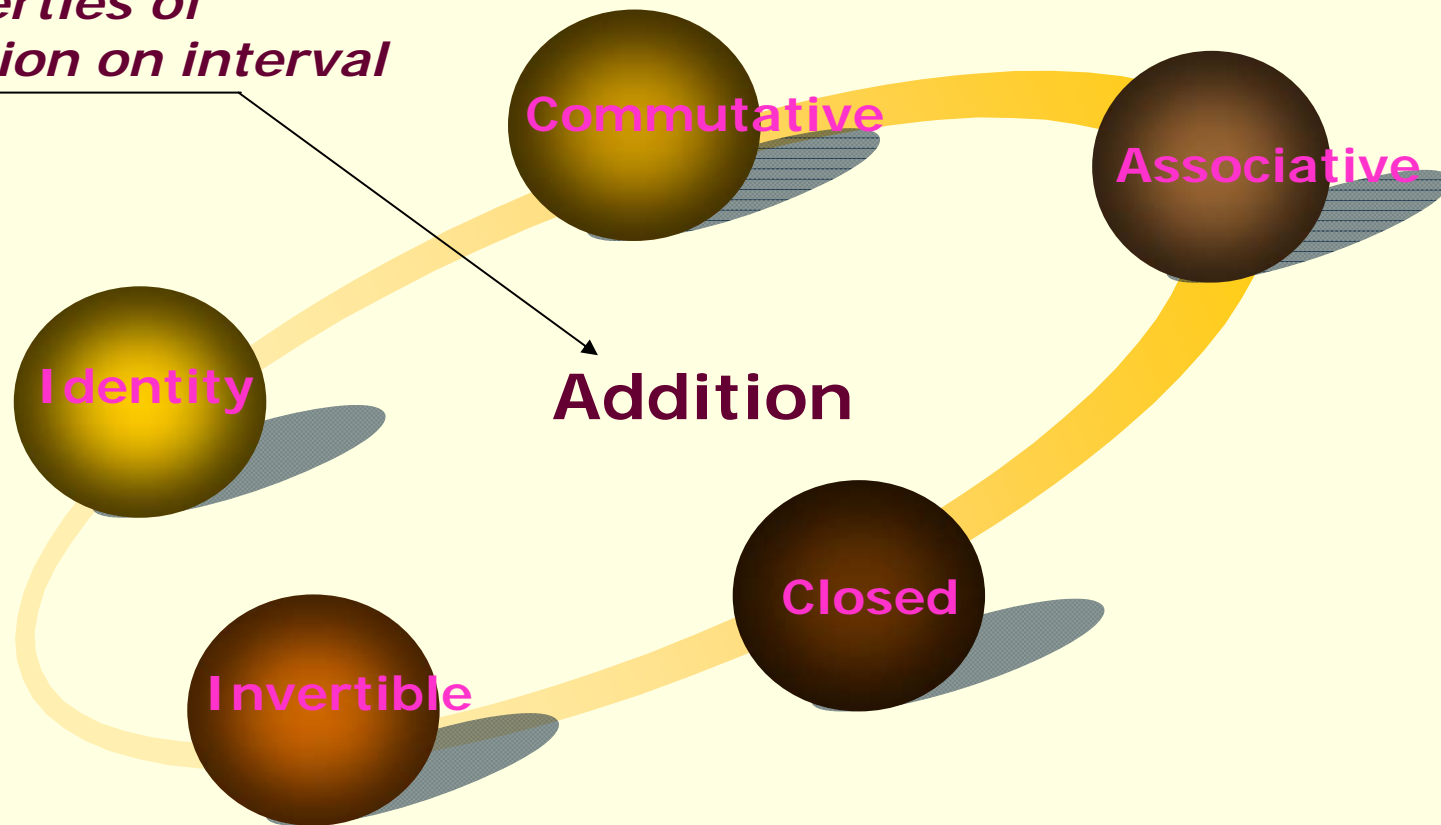 - Definition: add a interval to other interval and produce a new interval
  - Be a binary operation
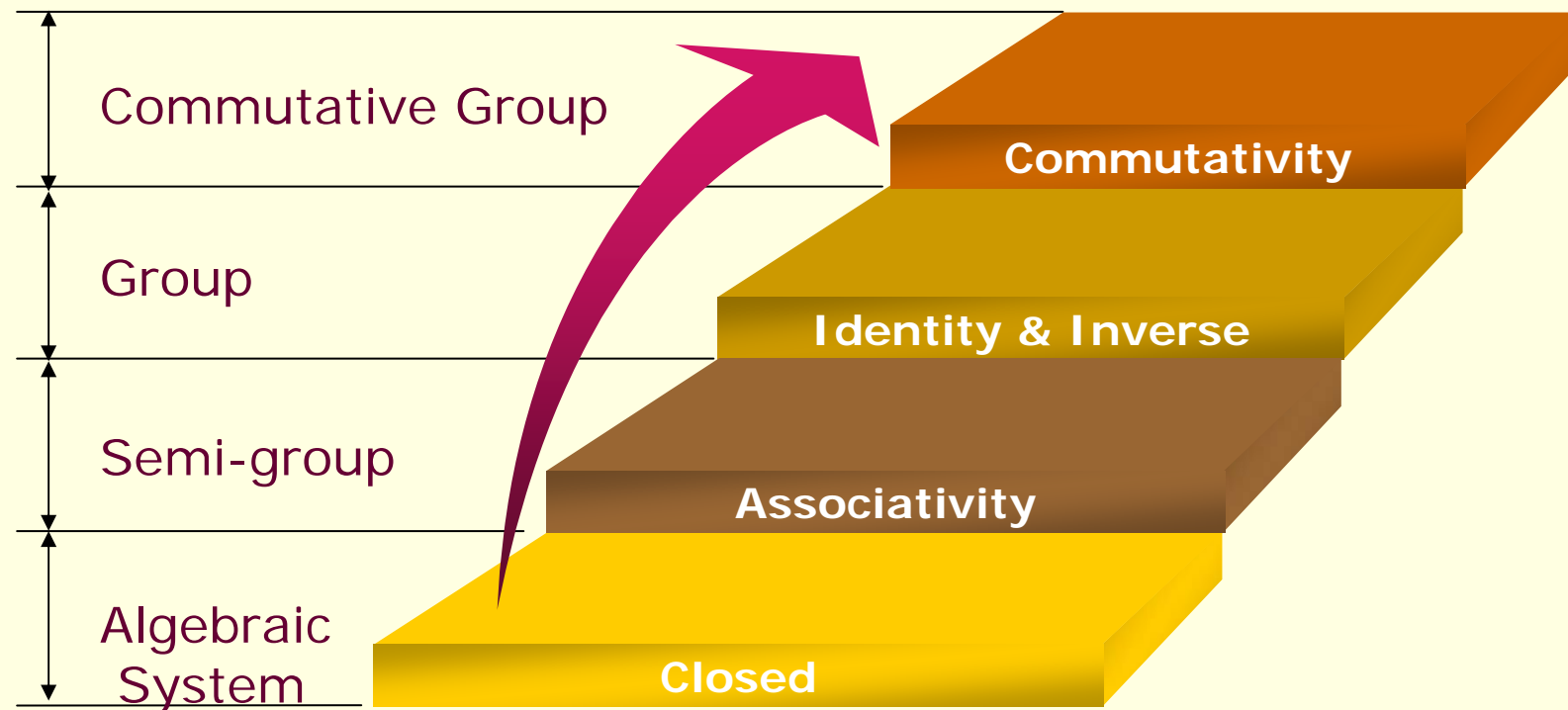 - Abbreviated to addition-interval

❖ Properties:
  - Closed: S is closed under addition
  - Associative: S satisfies associative law under addition
  - Commutative: S has commutativity under this operation
  - Identity:
    - system consisting of S and operation addition has identity
    - Called zero-interval
  - Inverse:
    - system consisting of S and operation addition has inverse for each element
    - Inverse of some interval equals zero-interval subtracts this interval

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Properties of Addition

Properties of Addition on interval

Commutative

Associative

Identity

Addition

Closed

Invertible

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Group with Addition

Commutative Group

Group

Semi-group

Algebraic System

Commutativity

Identity & Inverse

Associativity

Closed

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Multiplication on interval

❖ Conception
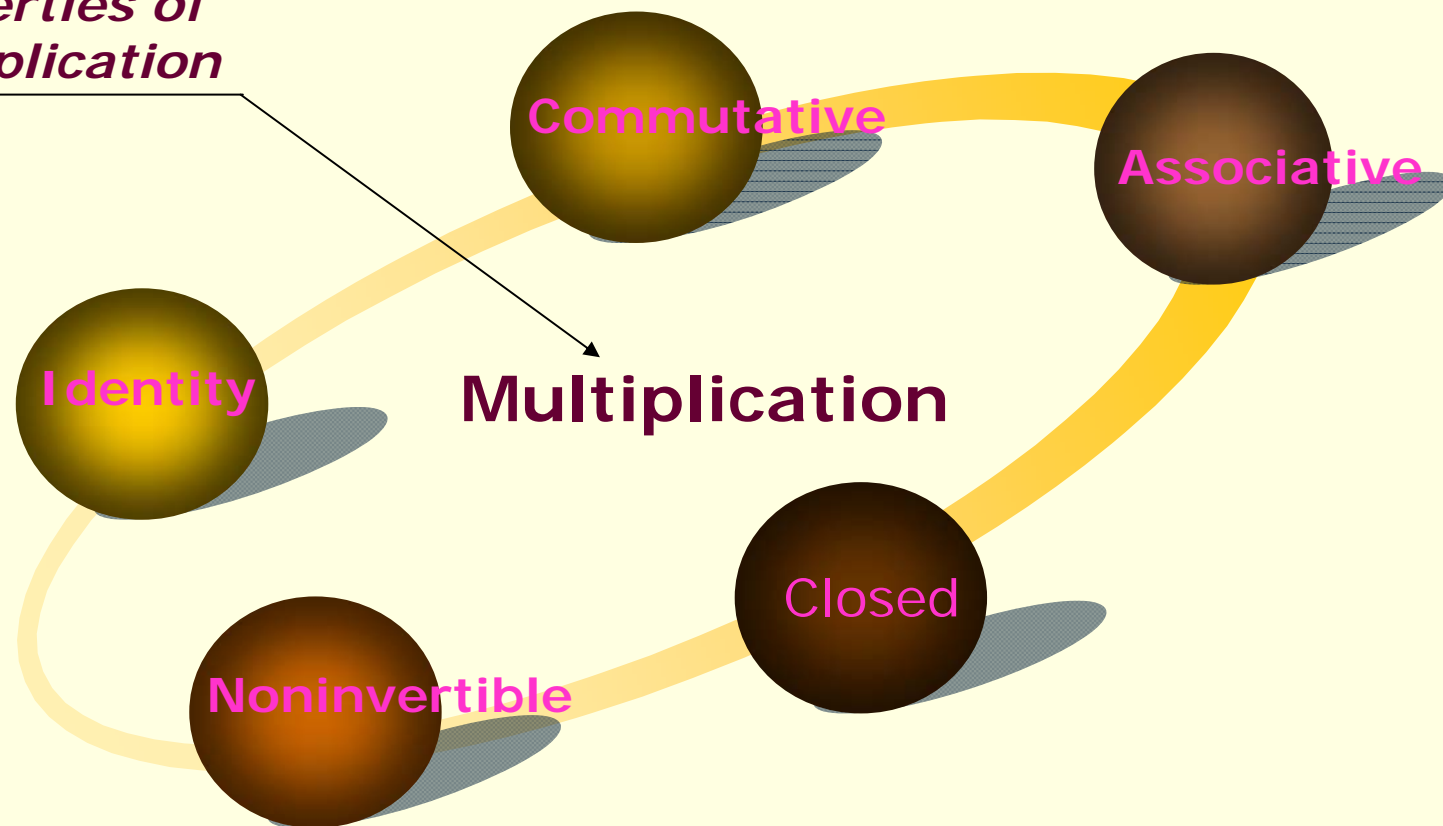 ᷎ Definition: multiply a interval by other interval and produce a new interval
  ᷎ Be a binary operation as addition
  ᷎ Abbreviated to multiplication-interval
❖ Properties:
  ᷎ Closed: S is closed under multiplication
  ᷎ Associative: S satisfies associative law under multiplication
  ᷎ Commutative: S is commutative under this operation
  ᷎ Identity:
    ✂ System consisting of S and operation multiplication has identity
    ✂ Called one-interval
  ᷎ Noninvertible
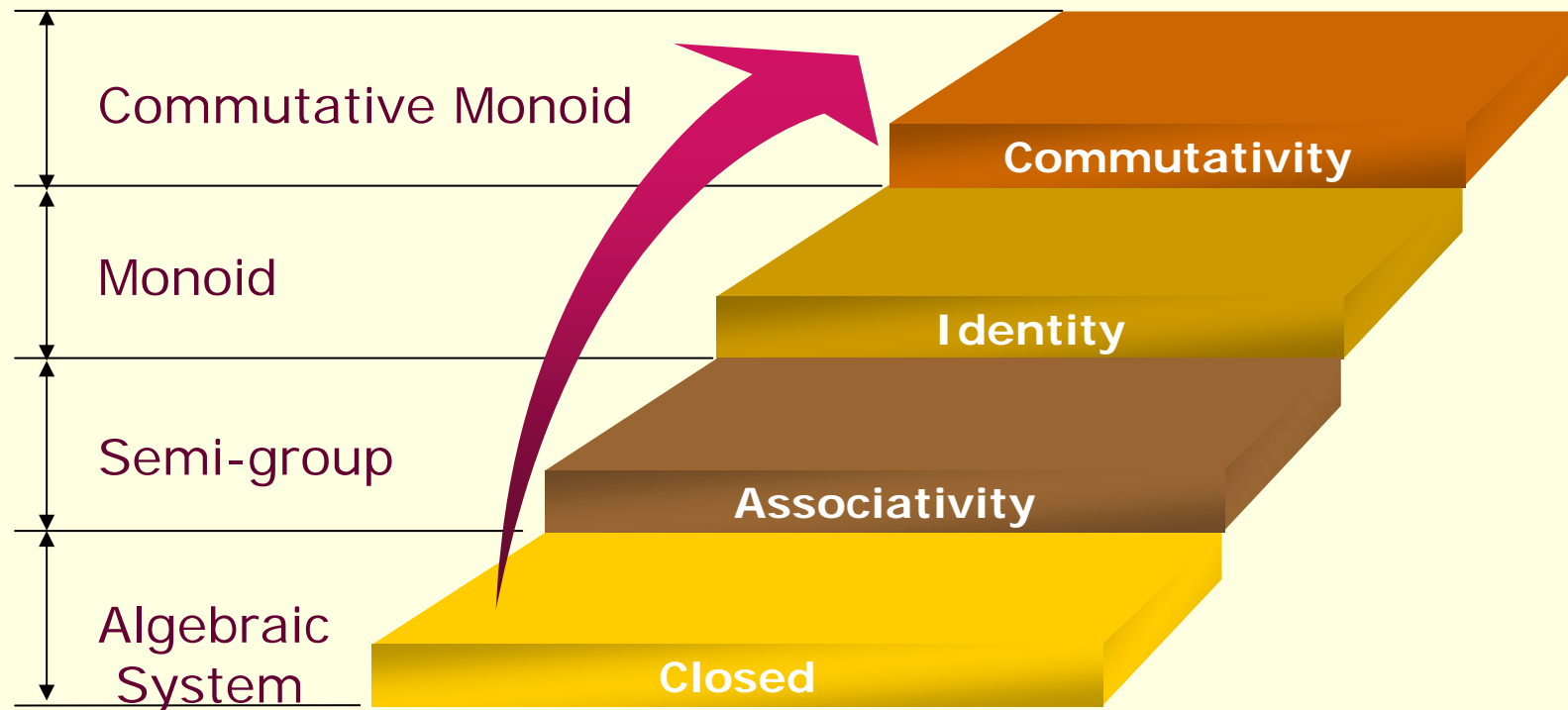   ✂ System consisting of S and operation addition has not inverse for some element

Properties of Multiplication

Properties of Multiplication

Commutative

Associative

Identity

Multiplication

Closed

Noninvertible

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Monoid with Multiplication

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Ring on S

- ❖ Conception
  - ⚬ Definition: system consisting of interval set S and operations of addition and multiplication
  - ⚬ Components
    - ✂ Interval set S
    - ✂ Operations on S: addition-interval, multiplication-interval
- ❖ Properties
  - ⚬ Inner algebraic system
    - ✂ Commutative Group with addition : Consisting of S and operation addition
    - ✂ Commutative semi-Group of multiplication : Consisting of S and operation multiplication
  - ⚬ Distributivity:
    - ✂ Operation addition is distributive by operation multiplication
    - ✂ Make result of operation on intervals unambiguous